

> restart

> Ecua := diff(y(x, t), x\$2) = c^2 * diff(y(x, t), t\$2)

$$Ecua := \frac{\partial^2}{\partial x^2} y(x, t) = c^2 \left(\frac{\partial^2}{\partial t^2} y(x, t) \right) \quad (1)$$

> EcuaUno := subs(c^2 = 1, Ecua)

$$EcuaUno := \frac{\partial^2}{\partial x^2} y(x, t) = \frac{\partial^2}{\partial t^2} y(x, t) \quad (2)$$

MEDIANTE EL MÉTODO DE SEPARACIÓN DE VARIABLES

> EcuaDos := eval(subs(y(x, t) = F(x) * G(t), EcuaUno))

$$EcuaDos := \left(\frac{d^2}{dx^2} F(x) \right) G(t) = F(x) \left(\frac{d^2}{dt^2} G(t) \right) \quad (3)$$

> EcuaSep := lhs(EcuaDos) / (F(x) * G(t)) = rhs(EcuaDos) / (F(x) * G(t))

$$EcuaSep := \frac{\frac{d^2}{dx^2} F(x)}{F(x)} = \frac{\frac{d^2}{dt^2} G(t)}{G(t)} \quad (4)$$

> EcuaXalfa := lhs(EcuaSep) = alpha

$$EcuaXalfa := \frac{\frac{d^2}{dx^2} F(x)}{F(x)} = \alpha \quad (5)$$

> EcuaTalfa := rhs(EcuaSep) = alpha

$$EcuaTalfa := \frac{\frac{d^2}{dt^2} G(t)}{G(t)} = \alpha \quad (6)$$

> CondFrontera := y(0, t) = 0, y(1, t) = 0

$$CondFrontera := y(0, t) = 0, y(1, t) = 0 \quad (7)$$

> CondIni := y(0, 0) = (a / (L/2)) * x, y(1, 0) = (2 * a) - (a / (L/2)) * x : CondIni[1]; CondIni[2]

$$y(0, 0) = \frac{2 a x}{L}$$

$$y(1, 0) = 2 a - \frac{2 a x}{L} \quad (8)$$

> DerCondIni := D[t](y(x, 0)) = 0

$$DerCondIni := D_t(y(x, 0)) = 0 \quad (9)$$

> L := 1; r := 5 / 1000

$$L := 1$$

(10)

$$r := \frac{1}{200} \quad (10)$$

> CondIni

$$y(0, 0) = 2 a x, y(1, 0) = -2 a x + 2 a \quad (11)$$

Para alfa = 0

> EcuaXcero := subs(alpha=0, EcuaXalfa)

$$EcuaXcero := \frac{\frac{d^2}{dx^2} F(x)}{F(x)} = 0 \quad (12)$$

> EcuaTcero := subs(alpha=0, EcuaTalfa)

$$EcuaTcero := \frac{\frac{d^2}{dt^2} G(t)}{G(t)} = 0 \quad (13)$$

> SolXcero := dsolve(EcuaXcero)

$$SolXcero := F(x) = c_1 x + c_2 \quad (14)$$

> ComprobarUno := subs(x=0, rhs(SolXcero)=0)

$$ComprobarUno := c_2 = 0 \quad (15)$$

> ComprobarDos := subs(x=1, c_2=0, rhs(SolXcero)=0)

$$ComprobarDos := c_1 = 0 \quad (16)$$

> F(x) = 0

$$F(x) = 0 \quad (17)$$

> y(x, t) = 0

$$y(x, t) = 0 \quad (18)$$

Para alfa positiva

> EcuaXpos := subs(alpha=β², EcuaXalfa)

$$EcuaXpos := \frac{\frac{d^2}{dx^2} F(x)}{F(x)} = \beta^2 \quad (19)$$

> SolXpos := dsolve(EcuaXpos)

$$SolXpos := F(x) = c_1 e^{-\beta x} + c_2 e^{\beta x} \quad (20)$$

> SistemaPos := eval(subs(x=0, rhs(SolXpos)=0)), eval(subs(x=1, rhs(SolXpos)=0)) :
SistemaPos[1]; SistemaPos[2]

$$c_1 + c_2 = 0$$

$$c_1 e^{-\beta} + c_2 e^{\beta} = 0 \quad (21)$$

> ParaPos := solve({SistemaPos}, {c₁, c₂})

$$ParaPos := \{c_1 = 0, c_2 = 0\} \quad (22)$$

> F(x) = 0; y(x, t) = 0

$$F(x) = 0$$

$$y(x, t) = 0 \quad (23)$$

Para alfa negativa

$$> \text{EcuaXneg} := \text{subs}(\text{alpha} = -\beta^2, \text{EcuaXalfa})$$

$$\text{EcuaXneg} := \frac{\frac{d^2}{dx^2} F(x)}{F(x)} = -\beta^2 \quad (24)$$

$$> \text{SolXneg} := \text{dsolve}(\text{EcuaXneg})$$

$$\text{SolXneg} := F(x) = c_1 \sin(\beta x) + c_2 \cos(\beta x) \quad (25)$$

$$> \text{ComprobarCinco} := \text{simplify}(\text{subs}(x=0, \text{rhs}(\text{SolXneg})=0))$$

$$\text{ComprobarCinco} := c_2 = 0 \quad (26)$$

$$> \text{ComprobarSeis} := \text{subs}(c_2=0, \text{beta}=n \cdot \text{Pi}, \text{rhs}(\text{SolXneg})=0); c_1 \neq 0$$

$$\text{ComprobarSeis} := c_1 \sin(n \pi x) = 0$$

$$c_1 \neq 0 \quad (27)$$

$$> \text{EcuaXfinal} := \text{subs}(\text{alpha} = -(n \cdot \text{Pi})^2, \text{EcuaXalfa})$$

$$\text{EcuaXfinal} := \frac{\frac{d^2}{dx^2} F(x)}{F(x)} = -n^2 \pi^2 \quad (28)$$

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$$> \text{SolXfinal} := \text{subs}(c_2=0, c_1=1, \text{dsolve}(\text{EcuaXfinal}))$$

$$\text{SolXfinal} := F(x) = \sin(n \pi x) \quad (29)$$

$$> \text{EcuaTneg} := \text{subs}(\text{alpha} = -(n \cdot \text{Pi})^2, \text{EcuaTalfa})$$

$$\text{EcuaTneg} := \frac{\frac{d^2}{dt^2} G(t)}{G(t)} = -n^2 \pi^2 \quad (30)$$

$$> \text{SolTneg} := \text{dsolve}(\text{EcuaTneg})$$

$$\text{SolTneg} := G(t) = c_1 \sin(n \pi t) + c_2 \cos(n \pi t) \quad (31)$$

$$> \text{SolGralNeg} := y(x, t) = \text{rhs}(\text{SolXfinal}) \cdot \text{rhs}(\text{SolTneg})$$

$$\text{SolGralNeg} := y(x, t) = \sin(n \pi x) (c_1 \sin(n \pi t) + c_2 \cos(n \pi t)) \quad (32)$$

$$> \text{SolGralFourier} := y(x, t) = \text{Sum}(\sin(n \cdot \text{Pi} \cdot x) \cdot (b[n] \cdot \cos(n \cdot \text{Pi} \cdot t) + a[n] \cdot \sin(n \cdot \text{Pi} \cdot t)), n=i \dots \text{infinity})$$

$$\text{SolGralFourier} := y(x, t) = \sum_{n=i}^{\infty} \sin(n \pi x) (b_n \cos(n \pi t) + a_n \sin(n \pi t)) \quad (33)$$

$$> \text{SolGralTotal} := \text{eval}(\text{subs}(t=0, \text{SolGralFourier}))$$

$$\text{SolGralTotal} := y(x, 0) = \sum_{n=i}^{\infty} \sin(n \pi x) b_n \quad (34)$$

$$\begin{aligned}
 & \text{> } b[n] := \text{subs} \left(\sin(n \cdot \text{Pi}) = 0, \cos(n \cdot \text{Pi}) = (-1)^n, \text{simplify} \left(\left(\frac{1}{\left(\frac{1}{2}\right)} \right) \cdot \text{int} \left(\left(\frac{\left(\frac{1}{100}\right)}{\left(\frac{1}{2}\right)} \cdot x \right. \right. \right. \right. \right. \\
 & \quad \left. \left. \left. \cdot \sin(n \cdot \text{Pi} \cdot x), x = 0 \dots \frac{5}{10} \right) \right) + \left(\frac{1}{\left(\frac{1}{2}\right)} \right) \cdot \text{int} \left(\left(\frac{2}{100} \right) - \frac{\left(\frac{1}{100}\right)}{\left(\frac{1}{2}\right)} \cdot x \right) \cdot \sin(n \cdot \text{Pi} \cdot x), x \right. \right. \\
 & \quad \left. \left. = \frac{5}{10} \dots 1 \right) \right) \right) \\
 & \quad b_n := \frac{2 \sin\left(\frac{n \pi}{2}\right)}{25 n^2 \pi^2} \tag{35}
 \end{aligned}$$

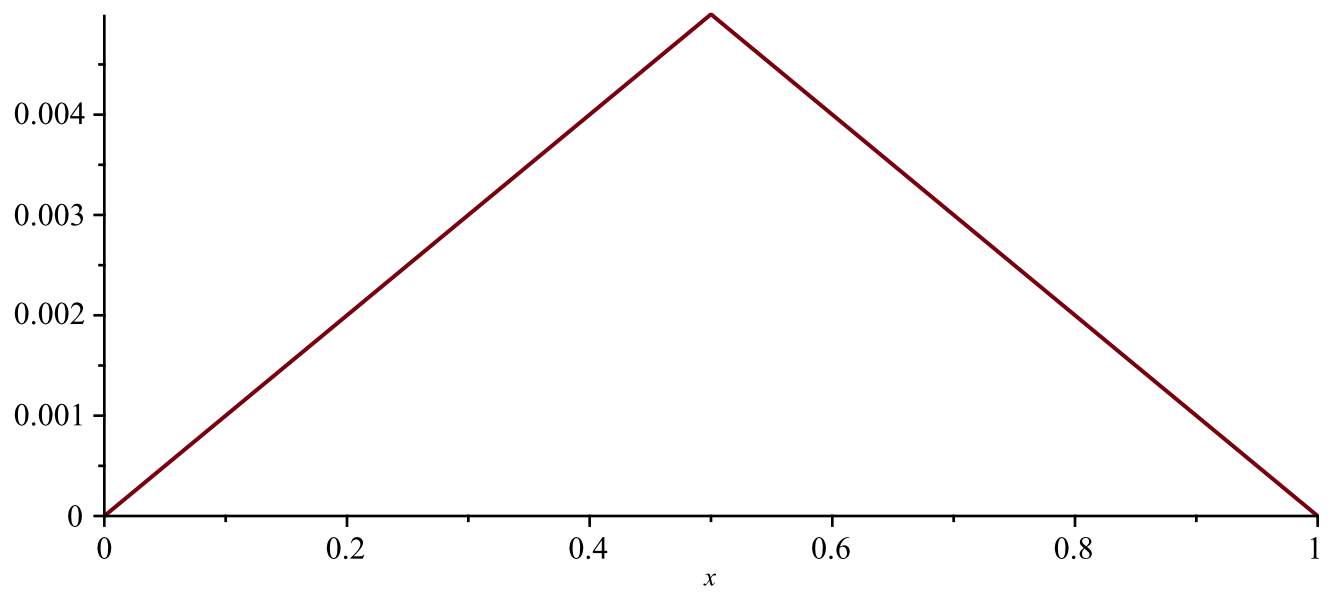
$$\begin{aligned}
 & \text{> } \text{isolate}(\text{eval}(\text{rhs}(\text{subs}(t=0, \text{diff}(\text{SolGralFourier}, t)))) = 0), a[n]) \\
 & \quad \sum_{n=i}^{\infty} \sin(n \pi x) a_n n \pi = 0 \tag{36}
 \end{aligned}$$

$$\begin{aligned}
 & \text{> } a[n] := 0 \\
 & \quad a_n := 0 \tag{37}
 \end{aligned}$$

$$\begin{aligned}
 & \text{> } \text{SolGralFourier} \\
 & \quad y(x, t) = \sum_{n=i}^{\infty} \frac{2 \sin(n \pi x) \sin\left(\frac{n \pi}{2}\right) \cos(n \pi t)}{25 n^2 \pi^2} \tag{38}
 \end{aligned}$$

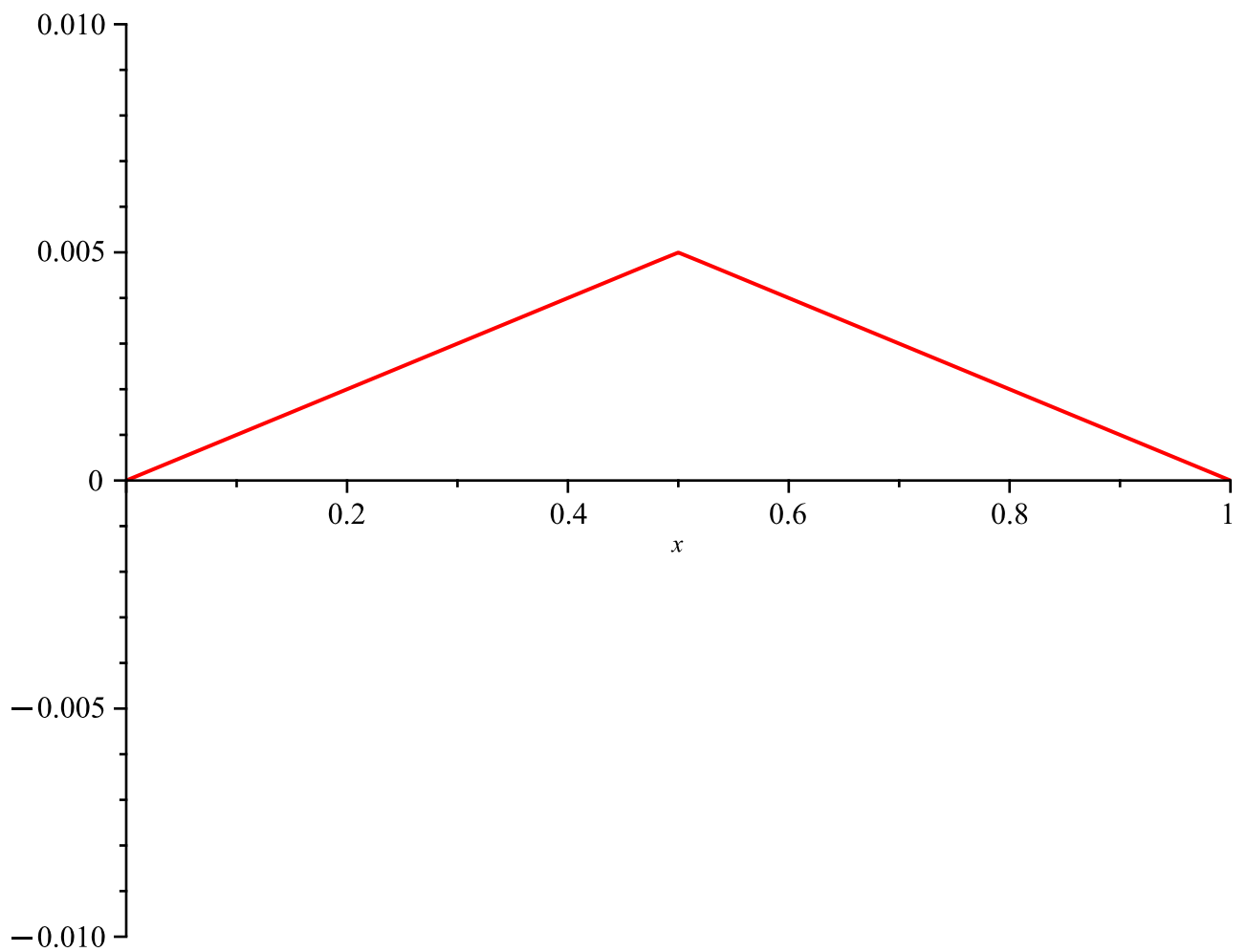
$$\text{> } \text{SolPart500} := y(x, t) = \text{sum} \left(\frac{\sin(n \pi x) \sin\left(\frac{n \pi}{2}\right) \cos(n \pi t)}{25 n^2 \pi^2}, n = 1 \dots 500 \right) :$$

$$\text{> } \text{plot}(\text{subs}(t=0, \text{rhs}(\text{SolPart500})), x = 0 \dots 1)$$



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> with(plots) :
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> animate(rhs(SolPart500), x = 0 .. 1, t = 0 .. 4, frames = 150, view = [0 .. 1, -0.01 .. 0.01])
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